

Department of Systems Innovation

2021 Online Written Examination: Problems Designed to Test Ability of Logical Thinking Field 2: Artificial intelligence, information science, and mathematical models

Question 2-1

You have N mails in your mailbox. Each mail has its priority for replying, which is denoted by $x \in (0, 1)$. At each time step, you either choose the mail which has the highest priority with probability p or randomly choose a mail with probability $1 - p$. You reply to the mail you have chosen and delete it from your mailbox. After replying, you receive a new mail whose priority x is drawn from a uniform distribution on the interval $(0, 1)$.

- (1) First, let $N = 2$ and $p = 1$. Second, let **A** and **B** denote two mails you initially have and let x_A and x_B be the priorities of these mails, respectively, in the ascending order (i.e. $x_A < x_B$). Find the probability if the duration (the number of time steps) until you reply to the mail **A** is t .
- (2) Next, let $N \gg 1$ and $p = 1/2$. Consider the case that the above process (i.e. receiving and replying to mails) is repeated many times. Let $f(x)$ be the distribution function of the priorities x of the mails in your mail box and let $g(t)$ be the distribution function of the duration t (i.e. the number of time steps needed to reply to each mail). Then, describe the characteristic features of these two distribution functions $f(x)$ and $g(t)$, and sketch their curves.

Question 2-2

There are $m + n$ data points $\{\mathbf{x}_1, \dots, \mathbf{x}_{m+n}\}$, which consist of d -dimensional real values ($d \geq 3$). Note that $\{\mathbf{x}_1, \dots, \mathbf{x}_m\}$ belong to the class C_1 , and $\{\mathbf{x}_{m+1}, \dots, \mathbf{x}_{m+n}\}$ belong to the other class C_2 .

Assuming $m \gg 1$ and $n \gg 1$, answer the following questions regarding the method for determining the class of a new data point \mathbf{x}_0 . If needed, you can draw figures.

- (1) Give two or more examples of the methods for determining the class of data points and describe the features of the methods.
- (2) Choose one method from your examples given in (1) and describe its algorithm using such as equations, pseudo-codes*, or flow-charts.
- (3) Explain how to evaluate the accuracy of the method described in (2).

*Pseudo-code is an informal description of the operating principle of a computer program or algorithm. Below is an example pseudo-code to give an order to study unless being sleepy.

```
if isSleepy == true:  
    Go to bed  
else:  
    Study
```