Examinee Number ____________________

2020 Entrance Examination
Department of Systems Innovation /
Department of Nuclear Engineering and Management /
Department of Technology Management for Innovation,
Graduate School of Engineering, The University of Tokyo

Mathematical Problems Designed to Test Ability to Think Logically

Monday, August 26, 2019

13:00 - 15:30

Documents distributed:

1. This booklet
2. One answer sheet for Mathematical Problems Designed to Test Ability to Think Logically
3. One problem booklet for 2020 The Graduate School Entrance Examination, Mathematics
4. Two answer sheets for 2020 The Graduate School Entrance Examination, Mathematics

General instructions:

- ○ Answers should be written in Japanese or English.
- ○ Do not open any problem booklets until the start of the examination is announced.
- ○ Confirm that all documents above are correctly distributed. Notify your proctor if you find any missing items.
- ○ Notify your proctor if you find any printing or production errors.
- ○ Write your examinee number in the designated places of all the documents distributed.
- ○ Do not take any documents distributed with you after the examination.
- ○ **Answer four problems out of the six given in this booklet on the answer sheet (Document 2).** Write your answer including your solution process. **Fill in the problem numbers in the designated places on the answer sheet (Document 2) and also circle the problem numbers you selected (P1, P2, …, P6) on the sheet. You are not allowed to choose more than four problems.**

- ○ **Answer two problems out of the six given in Document 3** (2020 The Graduate School Entrance Examination, Mathematics) **on the answer sheets** (Document 4)**. You are not allowed to answer more than two problems.**

Problem 1

Find the maximum and the minimum values of the function $f(x)=\dfrac{6ax^2+10ax+1}{3x+5}$ for the interval $0\le x\le 1$, under the condition of the constant a: $\dfrac{3}{128}\le a\le\dfrac{3}{50}$.

Draft Sheet

Problem 2

Find the positive integer numbers a and b that satisfy the following equation. Describe your solution process.

$$\frac{419}{999} = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{a+\cfrac{1}{b}}}}}}}}$$

Draft Sheet

Problem 3

Atmospheric pressure at a certain altitude is equal to the weight of air above the horizontal plane at the altitude divided by its area. At the sea level, the measured atmospheric pressure is 1.0 kgf/cm^2 *, and the measured weight of 1.0 ℓ of air is 1.0 gf.

Assume that the density of air is described using the exponential function of altitude by neglecting temperature change with altitude. Calculate the ratio of the atmospheric pressure at the altitude of 10 km to that at the sea level. Answer with two significant digits. Use 2.7 as the base of the natural logarithm if necessary.

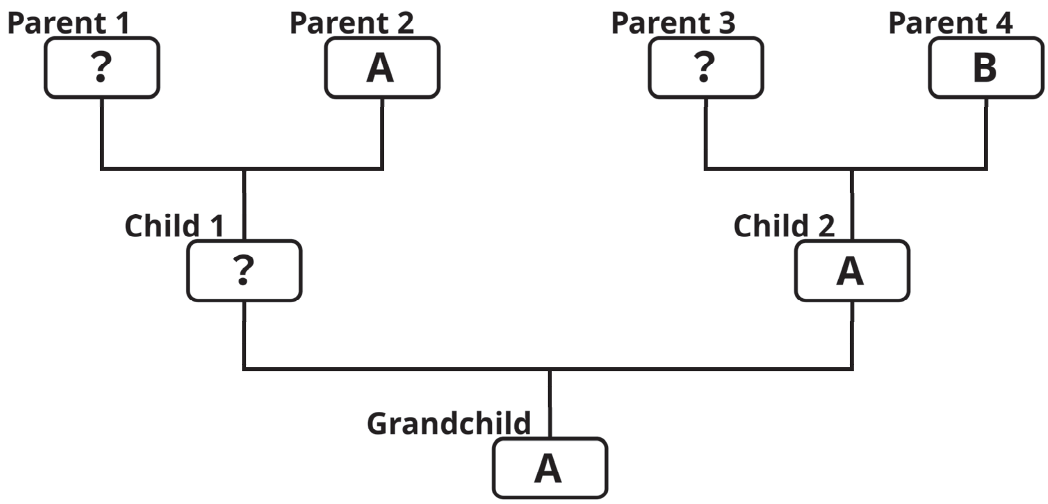
* 1 kgf = 1000 gf = 9.8 N

Draft Sheet

Problem 4

Consider two genes (alleles) ● and ○ whose proportions among the entire population are equal. Each person in this population has one of the following genotypes* : ●●, ●○, or ○○. A child receives one gene from each parent randomly. For example, for children from parents both having a genotype of ●○, the probabilities to have the genotypes ●●, ●○, and ○○ are 1/4, 1/2, and 1/4, respectively. The genotypes ●● and ●○ express phenotype** A, and the other genotype ○○ expresses phenotype B.

When the phenotypes of three generations are partially known as shown in the figure below, estimate the probability that the genotype of parent 1 is ●●.



* Genotype is the combination of genes of a living thing.

** Phenotype is the observable properties of a living thing, resulting from the genotype.

Draft Sheet

Problem 5

Answer the following questions.

(1) Two players play rock-paper-scissors games*. In case the winner is not determined in one game, they will do another game until one of the players becomes the winner. Calculate the probability that the winner is determined at the n-th game.

(2) Three players play rock-paper-scissors games to determine a winner. In case one of the players loses in the game, the one does not take part in the following games. In case the winner is not determined in the game, the players will do another game. Calculate the probability that the winner is determined at the n-th game.

* Rock-paper-scissors game

Rock-paper-scissors is a game with the three shapes of a hand; "rock", "paper", and "scissors". Every player selects one of the shapes simultaneously. Rock wins scissors, scissors wins paper, and paper wins rock. In the case that all the players select the same shape or that all the three shapes are selected, the game is "draw"; no winner, no loser.

If it is played between two players, one game has only two possible outcomes: (i) a win for one player and a loss for the other, or (ii) a draw.

If it is played among three players, one game has three possible outcomes: (i) a win for one player and a loss for the other two, (ii) a loss for one player and a win for the other two, or (iii) a draw.

Draft Sheet

Problem 6

Consider the circle of radius 1 with its center at the origin in the x-y plane. Answer the following questions.

(1) Find a rational point (a point whose x- and y-coordinates are both rational numbers) on the circumference, under the condition that $x > 0$ and $y > 0$.

(2) Consider a straight line which passes through the point on the x-axis, P $(-1, 0)$, and the point on the y-axis, T $(0, t)$ (where $0 < t < 1$). The straight line crosses the circumference at the two different points; P and Q. Express the coordinates of Q using t.

(3) There are an infinite number of rational points on the circumference. Explain the reason.

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