Examinee Number _____

2018 Entrance Examination
Department of Systems Innovation /
Department of Nuclear Engineering and Management /
Department of Technology Management for Innovation,
Graduate School of Engineering, The University of Tokyo

Mathematical Problems Designed to Test Ability to Think Logically

Monday, August 28, 2017

13:00 - 15:30

Documents distributed:

1. One problem booklet for 2018 Entrance Examination, Mathematical Problems Designed to Test Ability to Think Logically (this booklet)
2. One answer sheet for 2018 Entrance Examination, Mathematical Problems Designed to Test Ability to Think Logically
3. One problem booklet for 2018 The Graduate School Entrance Examination, Mathematics
4. Two answer sheets for 2018 The Graduate School Entrance Examination, Mathematics

General instructions:

○ Answers should be written in Japanese or English.
○ Do not open any problem booklets until the start of the examination is announced.
○ Confirm that all documents above are correctly distributed. Notify your proctor if you find any missing items.
○ Notify your proctor if you find any printing or production errors.
○ Write your examinee number in the designated places of Document 1 (this booklet), Document 2 (answer sheet), Document 3 (2018 The Graduate School Entrance Examination, Mathematics), and Document 4 (answer sheets), respectively.
○ Do not take any items distributed with you after the examination.
○ **Answer four problems out of the six given in this booklet on the answer sheet (Document 2).** Write your answer including your solution process. **Fill in the problem numbers in the designated places at the answer sheet (Document 2) and also circle the problem numbers you selected (P1, P2, …, P6) on that sheet. You are not allowed to choose more than four problems.**

○ **Answer two problems out of the six given in Document 3** (2018 The Graduate School Entrance Examination, Mathematics) **on the answer sheets** (Document 4)**. You are not allowed to answer more than two problems.**

Problem 1

Consider that all edges of a regular tetrahedron and all edges of a regular octahedron are in contact with a sphere with the radius *r*.

Find the ratio of the volume of the regular tetrahedron to the volume of the regular octahedron.

You may use the following relationship between the volume V of a regular tetrahedron and the edge length a.

$$V = \frac{\sqrt{2}}{12} a^3$$

Draft Sheet

Problem 2

Find all sets of integer (x, y) that satisfy $2x^2 - 2xy + y^2 + 2x - 2y = 0$.

Draft Sheet

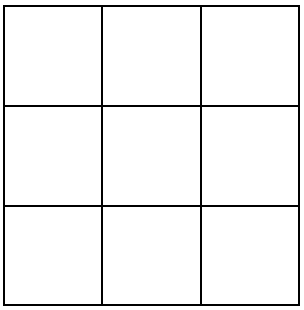
Problem 3

Alice and Bob play a game with 50 balls in a box. First, from the box, Alice takes out the balls in any number, ranging from 1 to 3. Then, Bob takes out the balls in any number, ranging from 1 to 3. In this way, Alice and Bob take out 1 to 3 balls alternately. The number of balls taken out can be changed each time. The player who takes out the last ball is the winner of the game. There is a way for Alice to win this game invariably. To win the game, how many balls should Alice take out first? Find the number of balls and describe your reasoning.

Draft Sheet

Problem 4

Consider a square grid with 3 × 3 cells as the figure below. Each cell of the square grid should have a distinct integer from 1 to 9. The sum of the numbers in each row, column and diagonal is equal to 15. Find the number in the central cell and describe your reasoning.



Draft Sheet

Problem 5

Find the number of ways to make change for a 1000 yen bill using 500 yen coin(s), 100 yen coin(s), 50 yen coin(s), and 10 yen coin(s). You may not use all four kinds of coins.

Draft Sheet

Problem 6

The calculation below shows addition in octal number system. R, I, S, K, C, A, and H should be any integer from 0 to 7. Each letter corresponds to a different number. Find one-to-one correspondence between the integers and the letters.

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   R I S K
+  R I S K
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 C R A S H
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