Examinee Number ____________________

2016 Entrance Examination Test and Answer Sheets
Department of Systems Innovation /
Department of Nuclear Engineering and Management /
Department of Technology Management for Innovation,
Graduate School of Engineering, The University of Tokyo

Mathematical Problems Designed to Test Ability to Think Logically

Monday, August 31, 2015 13:00 - 15:30

Notice:

1. Do not open this booklet until the start of the examination.

2. If you find missing or badly printed pages, please notify the supervisor.

3. Write your examinee number in the blank at the top of this page. Do not write your examinee number or name anywhere else in this booklet.

4. Write the answers including the outline of your solutions below each question.

5. Blank sheets of paper for calculation are distributed separately. After the examination, they must be returned.

6. This test has 15 questions in total. Choose and answer 12 questions out of 15.
Check the question numbers of the questions you have selected in the table below.
Please note that you are not allowed to answer more than 12 questions.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Q. 1

Find the coefficient of x^4yz^3 in the expansion of $(x+y+z)^8$.

Q. 2

Find the number of integers from 2000 to 3000 which are coprime to 3600.

Q. 3

Find the minimum value of x such that the following expression is a real number. Note that x is also a real number.

$$\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}}$$

Q. 4

The digit in the ones place of k^2 is 5 and the digit in the tens place of k^2 is 2. Obtain all possible digits in the hundreds place of k^2. Note that the integer $k \geq 10$.

Q. 5

A$(2,\ 3,\ 2)$, B$(5,\ 0,\ 5)$ and C$(4,\ -5,\ 0)$ are three points in the Cartesian coordinate system.

(1) Derive the equation of the plane containing all the points A, B and C.

(2) Find the coordinates of the center of the circle passing through all the points A, B and C.

Q. 6

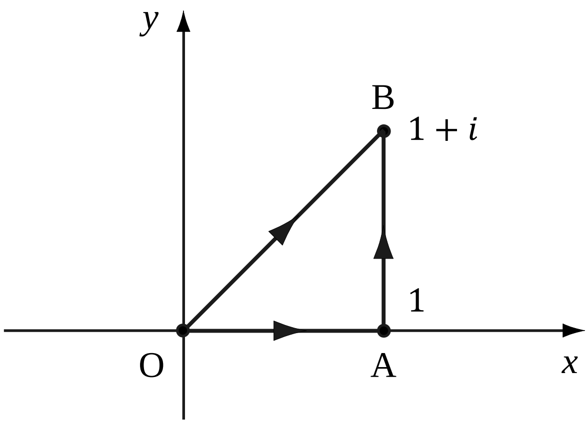
Three players A, B, and C are participating in a championship. In the first game, A plays against B. In the second and subsequent games, the previous winner plays against the remaining player. If any of the players wins two consecutive games, she or he becomes the champion. Games are repeated until the champion is determined. Note that the probability for each player to win a game is always 50%.

(1) Find the probability that the champion is determined within 3 games.

(2) Find the expected number of games for the champion to be determined.

Q. 7

Consider the function $f(z) = \bar{z} = x - iy$ with the complex variable $z = x + iy$. Let A be 1 and B be $1 + i$ on the complex plane, where i is the imaginary unit.



(1) Find the definite integral of $f(z)$ from O to B along OB.

(2) Find the definite integral of $f(z)$ from O to B along OAB.

Q. 8

When the sum of the three-digit integer CBC and the two-digit integer AB equals the three-digit integer DBE, find the greatest three-digit integer DBE. Note that A, B, C, D and E are different one-digit integers from 1 to 9.

Q. 9

Consider a point $\mathrm{P}(x, y)$ in the xy-plane. The distance between P and the fixed point $(1, 0)$ is a times the distance between P and the line $x = -1$, where $a > 0$.

(1) Find the condition of a when the locus of P is a parabola.

(2) Find the condition of a when the locus of P is a hyperbola.

Q. 10

The approximate value of π can be obtained by using the perimeter of the regular *n*-polygon inscribed within a circle of diameter 1. As *n* increases, the perimeter monotonically increases and approaches asymptotically to π. Find the minimum value of the integer *n*, when the approximation of π is greater than 3.1. The values of cosine are given in the table below.

n	$\cos(360°/n)$
7	0.623490
8	0.707107
9	0.766044
10	0.809017
11	0.841254
12	0.866025
13	0.885456
14	0.900969
15	0.913545

Q. 11

Find the orthogonal matrix U and the constants α, β, γ, when the following quadratic equation:

$$5{x_1}^2 + 6{x_2}^2 + 5{x_3}^2 + 2x_1x_2 - 2x_2x_3 - 4x_3x_1 = 3$$

is converted to:

$$\alpha{X_1}^2 + \beta{X_2}^2 + \gamma{X_3}^2 = 1$$

by the variable transformation: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = U \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$.

Q. 12

Find the following limits.

(1) $\lim_{x\to\infty}(2^{2x+1}+2^x)^{\frac{1}{x}}$

(2) $\lim_{x\to\infty}\frac{2^x+3^x}{x!}$

Note that x is a natural number.

Q. 13

Calculate the area of the circle formed by the intersection of two spherical surfaces S_1 and S_2.

S_1: $x^2 + y^2 + z^2 = 1$

S_2: $x^2 + y^2 + z^2 - 4x - 4y + 2z = 0$

Q. 14

Find the 10-digit number “FADHEEHDAF” which is obtained by multiplication as shown below. Each letter corresponds to a different integer from 0 to 9.

$$
\begin{array}{cccccccccc}
 & & & & & A & B & C & A & B \\
\times & & & & & A & D & E & D & A \\
\hline
 & & & & F & F & F & F & F & F \\
 & & & G & G & G & G & G & G & \\
 & & A & A & A & A & A & A & & \\
 & G & G & G & G & G & G & & & \\
F & F & F & F & F & F & & & & \\
\hline
F & A & D & H & E & E & H & D & A & F
\end{array}
$$

Q. 15

Solve the following definite integral:

$$\int_0^1 \frac{x^a - 1}{\log x}\, dx\,,$$

where a is a positive constant.