Examinee Number _____

2019 Entrance Examination
Department of Systems Innovation /
Department of Nuclear Engineering and Management /
Department of Technology Management for Innovation,
Graduate School of Engineering, The University of Tokyo

Mathematical Problems Designed to Test Ability to Think Logically

Monday, August 27, 2018

13:00 - 15:30

Documents distributed:

1. One problem booklet for 2019 Entrance Examination, Mathematical Problems Designed to Test Ability to Think Logically (this booklet)
2. One answer sheet for 2019 Entrance Examination, Mathematical Problems Designed to Test Ability to Think Logically
3. One problem booklet for 2019 The Graduate School Entrance Examination, Mathematics
4. Two answer sheets for 2019 The Graduate School Entrance Examination, Mathematics

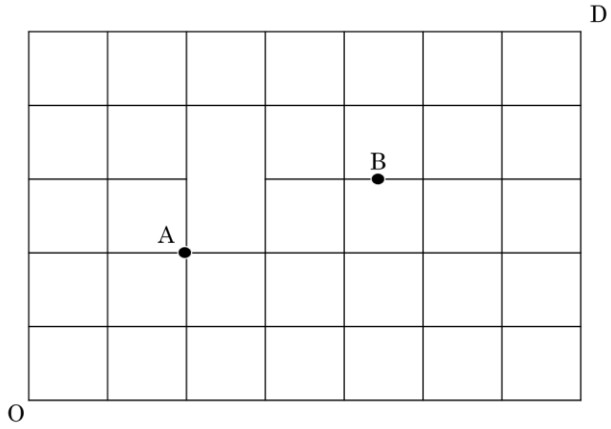
General instructions:

○ Answers should be written in Japanese or English.
○ Do not open any problem booklets until the start of the examination is announced.
○ Confirm that all documents above are correctly distributed. Notify your proctor if you find any missing items.
○ Notify your proctor if you find any printing or production errors.
○ Write your examinee number in the designated places of Document 1 (this booklet), Document 2 (answer sheet), Document 3 (2019 The Graduate School Entrance Examination, Mathematics), and Document 4 (answer sheets), respectively.
○ Do not take any items distributed with you after the examination.
○ **Answer four problems out of the six given in this booklet on the answer sheet (Document 2).** Write your answer including your solution process. **Fill in the problem numbers in the designated places at the answer sheet (Document 2) and also circle the problem numbers you selected (P1, P2, …, P6) on that sheet. You are not allowed to choose more than four problems.**

○ **Answer two problems out of the six given in Document 3** (2019 The Graduate School Entrance Examination, Mathematics) **on the answer sheets** (Document 4)**. You are not allowed to answer more than two problems.**

Problem 1

Count all shortest paths between O and D in the road shown below. Note that neither A nor B can be passed.



Draft Sheet

Problem 2

The cipher “022, 002, 022, 021, 211, 112” stands for “system”, and “020, 021, 110, 121, 002, 110” stands for “utokyo”. Decrypt “202, 120, 221, 120, 012, 011” and describe the reason for your answer.

Draft Sheet

Problem 3

Find the sums of the infinite series given in (1) and (2). Here, the following two equations can be used:

$$\sum_{k=1}^{\infty}\frac{1}{k^2}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6},\qquad \sum_{k=1}^{\infty}\frac{1}{k^4}=\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots=\frac{\pi^4}{90}.$$

(1) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$

(2) $\displaystyle\sum_{k=2}^{\infty}\sum_{l=1}^{k-1}\frac{1}{k^2l^2}=\frac{1}{2^21^2}+\frac{1}{3^21^2}+\frac{1}{3^22^2}+\frac{1}{4^21^2}+\frac{1}{4^22^2}+\frac{1}{4^23^2}+\cdots$

Draft Sheet

Problem 4

The area of the regular 12-sided polygon which is circumscribed about a circle A is four times as large as the area of the regular 24-sided polygon which is inscribed in a circle B. Find the ratio of the area of the circle A to the area of the circle B. Give your answer with three significant digits using $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$.

Draft Sheet

Problem 5

Three players, A, B, and C, played several one-on-one games, where the winner of a game played the next game and the loser was replaced with the other player. The first game was played by A and B. At the end, A, B, and C played 5, 8, and 5 games, respectively. No game was drawn. Find all players who might win the second game.

Draft Sheet

Problem 6

It took a cumulative total of 470158 minutes for a group of persons to assemble one robot. It also took a cumulative total of 247384 minutes for the same group to write a computer program. Find the number of persons in this group. For each task, all the persons in the group spent the same amount of time, and the amount of time was measured in minutes. Note that the number of persons in this group was from 20 to 200.

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