

Q1.

Region D is bounded by the line $y = x + 2$ and the parabola $y = x^2$.

(1) Obtain the area of D .

(2) Obtain the centroid (center of mass) of D .

Q2.

Positive integers are arranged as shown below. What is the number at the cell of $I=101$ and $J=50$?

	$I=1$	2	3	4	•
$J=1$	1	2	9	10	•
2	4	3	8	11	•
3	5	6	7	12	•
4	16	15	14	13	•
•	•	•	•	•	•

Q3.

Three real numbers 1 , A and B ($A < B$) can be ordered to form a geometric sequence and an arithmetic sequence. Find all pairs of A and B .

Q4.

Obtain the center of the circle passing through three points in Cartesian coordinates, $A(1, -2, 1)$, $B(3, 1, 7)$ and $C(2, 0, 6)$.

Q5.

Find the value of Z in the following calculation.

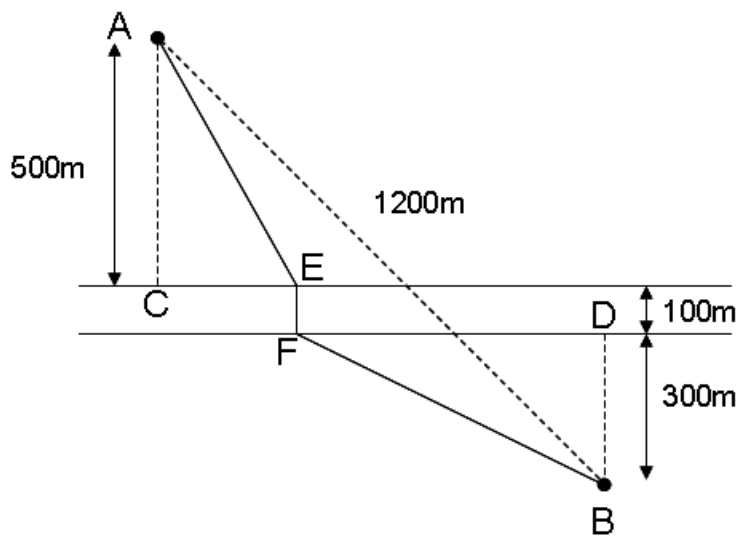
$$\begin{array}{r} \square \square \\ \times \square \square \\ \hline 2 \square \\ 6 \square \\ \hline 7 Z \square \end{array}$$

Q6.

When a three-digit number in nonary notation (base-9 number system) is expressed in septenary notation (base-7 number system), the order of the digits is reversed. Show this number in octal notation (base-8 number system).

Q7.

The following diagram shows that the width of the river, which lies between points A and B, is 100 m. It also shows that the distances between points A and C, between points B and D, and between points A and B are 500 m, 300 m, and 1200 m, respectively. A bridge EF orthogonal to the river should be constructed such that the route AEFB is the shortest. Find the distance between points C and E.



Q8.

Obtain the eigenvalues and the corresponding eigenvectors of a matrix

$$X = \begin{pmatrix} 1 & x & x & x \\ x & 1 & x & x \\ x & x & 1 & x \\ x & x & x & 1 \end{pmatrix},$$

where $x > 0$.

Q9.

Obtain the function $y(x)$ which satisfies the following differential equation

$$\frac{d^2y}{dx^2} = 2y^3 + 2y,$$

under the boundary conditions that $y(0) = 0$ and $\left. \frac{dy}{dx} \right|_{x=0} = 1$.

Q10.

Obtain the volume of the solid body which is expressed by the following inequality

$$|x + y + z| + |-x + y + z| + |x - y + z| + |x + y - z| \leq 4,$$

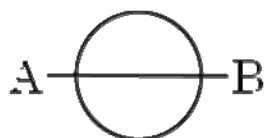
where x , y and z denote the Cartesian coordinates.

Q11.

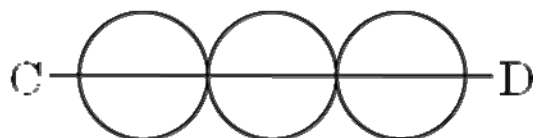
(1) In how many ways can the figure (a) be drawn with a single stroke from A to B?

(2) In how many ways can the figure (b) be drawn with a single stroke from C to D?

(a)



(b)



Q12.

Today's date is 8/31 (August 31). By merging the digits of the date (8 and 31), a number (831) is made.

- (1) In how many ways can 831 be expressed as a sum of two or more sequential positive integers?
- (2) Find the number of dates in a year which cannot be expressed as a sum of two or more sequential positive integers. If there is no such a date, answer as "zero" day. Note that 1/1 (January 1) makes 11, 2/10 (February 10) makes 210, and 10/1 (October 1) makes 101. Note also that, for example, the dates which make 123 are 1/23 (January 23) and 12/3 (December 3).

Q13.

A circular cylindrical cup of inside radius r and height h is completely full of water. Find the volume $V(\alpha)$ of remaining water when it is inclined by the angle α , assuming that the bottom of the cup is under the surface of water.

Q14.

Let $I_n = \int_0^{\pi/2} \sin^n x \, dx$. Evaluate I_4 and I_5 .

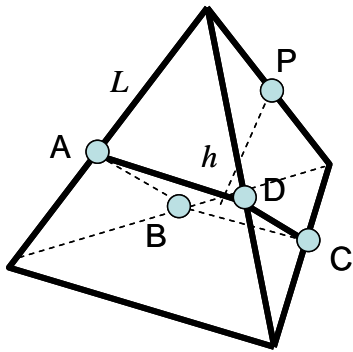
Q15.

Consider to tie cylindrical cans of radius r using a string. For example, the figure shows the case when two cans are tied. Obtain the shortest length of the string which ties seven cans.



Q16.

As shown in the figure, a square-based pyramid (P-ABCD) is made using the five midpoints on the edges of a regular tetrahedron of the edge-length L . Express the distance h between the base ABCD and the vertex P of the pyramid using L .



Q17.

Find the number of positive integers n of 1000 or less such that $n^{321} - 1$ are multiples of ten.

Q18.

Two taxi companies, **A** and **B**, operate in a city, and 25% of the taxis belong to **A** while 75% belong to **B**. One day, a taxi caused an accident. One witness said that the taxi belonged to **A**. Two types of people live in the city: the honest who does not tell a lie at all and the liar who always tell a lie. 70% of people in the city are the honest while 30% are the liar. The reliability that the witness correctly identifies the company is 80%. The probability that a taxi causes an accident is the same. What is the probability that the taxi belongs to **A**?

Q19.

When the complex number z satisfies

$$z^3 = -10 + 9\sqrt{3}i,$$

obtain $z\bar{z}$ and $z + \bar{z}$, where i is the imaginary unit, and \bar{z} is the complex conjugate of z .

Q20.

Initially, box **A** contains four black balls, and box **B** contains four white balls. We perform three successive ball exchanges. In each exchange, we pick simultaneously and at random a ball from each box and move it to the other box. What is the probability that at the end of the three exchanges box **A** contains two white balls?