

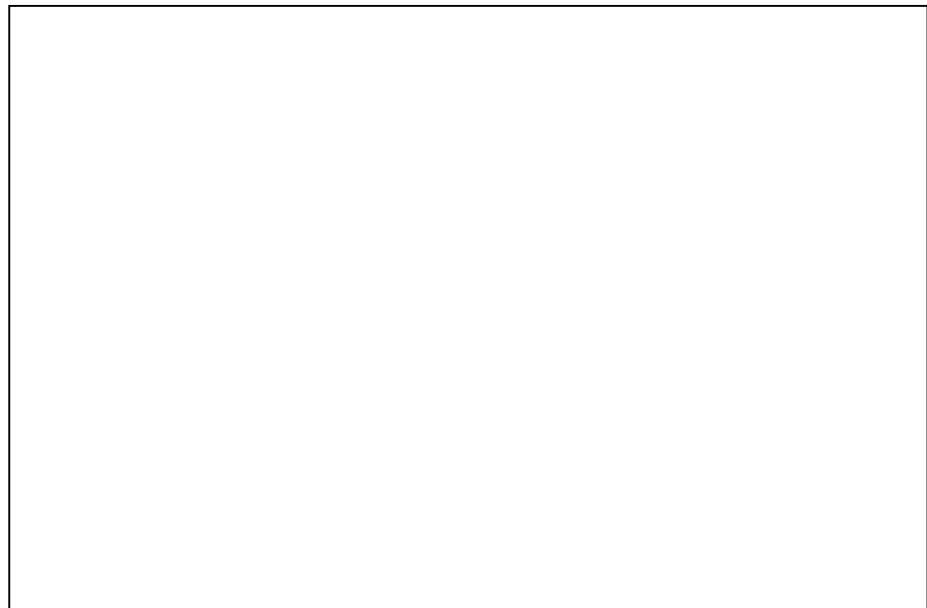
Q1.

Obtain A^n of the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix},$$

where n is a positive integer, i is the imaginary unit, and

$$\omega = \frac{-1 + \sqrt{3}i}{2}.$$



Q2.

When 9 letters, AAAABBCCC are aligned randomly, what is the probability that the symmetrical alignments, such as AABCCCBAA are generated?



Q3.

Obtain all combinations of the positive integer numbers l , m , and n that satisfy the following relational expression:

$$lmn = 2l + m + n, \quad l \geq m \geq n.$$



Q4.

One black ball and three white balls (a total of four balls) are included in each X and Y box. One ball is exchanged between these boxes in one trial. Obtain the probability of a specific state in which each box has one black ball and three white balls after the N^{th} trial.



Q5.

Consider a sphere of radius r . When the section of the sphere intersected by the $x - y$ plane ($z = 0$) is a circle of radius a , the section of the sphere intersected by the $y - z$ plane ($x = 0$) is a circle of radius b , and the section of the sphere intersected by the $z - x$ plane ($y = 0$) is a circle of radius c , find the distance between the origin of the coordinates and the center of the sphere.



Q6.

Evaluate

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx.$$



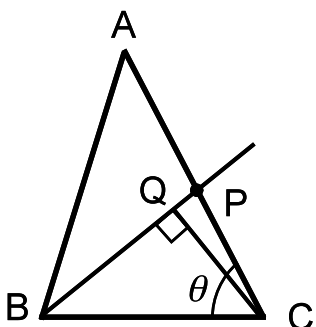
Q7.

Find the total area of the regions bounded by functions e^{-x} and $e^{-x} \sin x$ in the semi-infinite interval $0 \leq x < +\infty$.



Q8.

Consider a triangle ABC where $AC = 4$, $BC = 3$, $\angle ACB = \theta$, $\overrightarrow{CA} = \vec{a}$, and $\overrightarrow{CB} = \vec{b}$. The point P is the midpoint of the side AC , and a perpendicular line CQ is drawn from the point C to the straight line BP . Express the \overrightarrow{CQ} using θ , \vec{a} , and \vec{b} .



Q9.

Vectors \vec{a} , \vec{b} , and \vec{c} are shown in Fig. 1. Parallelepiped V_1 is constructed using these vectors, as shown in Fig. 2. Then hexahedron V_2 is constructed inside V_1 using the center points of faces and two vertices of V_1 , as shown in Fig.3. When $\vec{a} = (3, 1, 0)$, $\vec{b} = (2, 3, 1)$, and $\vec{c} = (1, 1, 3)$, calculate the volume of V_2 .

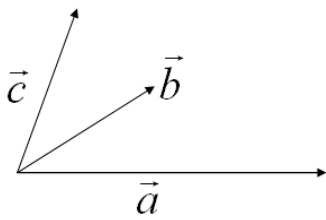


Fig. 1

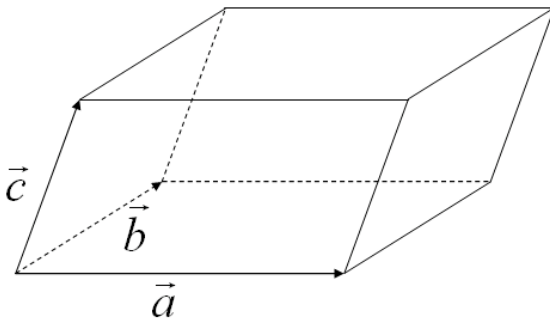


Fig. 2

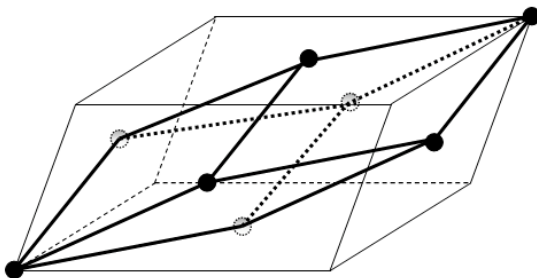


Fig. 3



Q10.

Obtain the area of the circle that is formed by the intersection of two spherical surfaces:

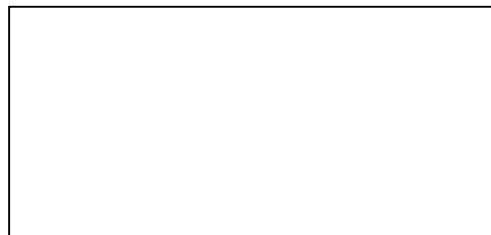
$$S_1: x^2 + y^2 + z^2 = 1,$$

$$S_2: x^2 + y^2 + z^2 + 4x - 4y + 2z + 1 = 0.$$



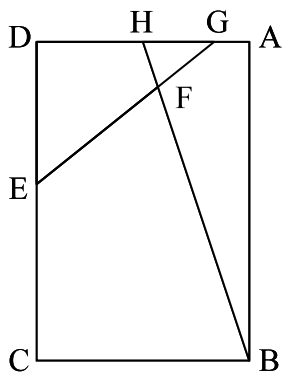
Q11.

A point P is chosen at random on the circle $x^2 + y^2 = 1$. The variable X denotes the distance of P from $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Find the mean of X .



Q12.

Find the area of the quadrangle EFBC, where ABCD is a rectangle and $AG = 1$, $GH = 2$, $HD = 3$, $DE = 4$, $EC = 5$.



Q13.

Find A and B when numbers 1 to 16 are put in each cell of the 4×4 matrix below such that the sums of 4 numbers in all directions (row, column, or diagonal) are identical.

4		15	B
5	11		
	7		12
A			13



Q14.

Evaluate

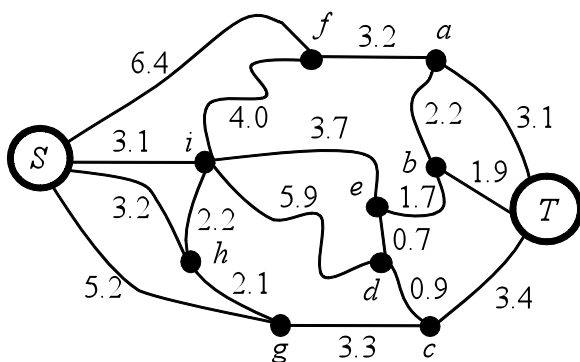
$$(\sqrt{3}i - 1)^0 + (-\sqrt{3}i - 1)^0,$$

where i is the imaginary unit.



Q15.

In the graph shown below, find the shortest path between S and T , where the numbers beside the branches indicate the length of the corresponding branches.



Q16.

Find the value of A in the following calculation.

$$\begin{array}{r} \square\square \\ \times \square\square 8 \\ \hline \square\square\square \\ 00 \\ \square\square \\ \hline 1\square 6 A\square \end{array}$$



Q17.

P and Q are defined as:

$$P = \sqrt{2 + 3\sqrt{2 + 3\sqrt{2 + 3\sqrt{2 + \dots}}}},$$

$$Q = a + \frac{2}{a + \frac{2}{a + \frac{2}{a + \dots}}}.$$

Suppose P and Q converge, and $a > 0$. When $P = Q$, find a .



Q18.

A fraction, which includes 4 numbers multiplied in the numerator and 3 numbers multiplied in the denominator, has a value of 1:

$$\frac{\square \cdot \square \cdot \square \cdot \square}{\square \cdot \square \cdot \square} = 1.$$

Use 7 numbers (2, 4, 8, 16, 32, 64, 128) for the numerator and the denominator. Find all combinations of 4 numbers in the numerator and 3 numbers in the denominator.



Q19.

How many natural numbers are there below 1000 that are multiples of 3 or that contain 3 in any digit of the number?



Q20.

Following two expressions represent words written in code:

(49, 75, 113, 126, 129): key 37 = labor,

(71, 45, 53, 67, 112, 82): key 31 = invest.

Break the code and find the word for:

(106, 112, 77, 107, 92, 71): key 29 = ???

