Examinee's No. ________________

Department of Geosystem Engineering

University of Tokyo
2008 Entrance Examination for the Master Course

Mathematics

August 27, 2007

13:00～15:30

Notice

1. Do not open this booklet until the start of the examination.
2. Write your examinee's number on the top of this sheet. Do not write your examinee's number or your name anywhere else.
3. There are 20 questions in total. Choose and answer 15 questions out of 20. Circle the numbers of the questions you have selected in the bottom row.
4. Write the outline of your solution below the question and put only the answer in the box.
5. Sheets for calculation are distributed separately.

Circle the number of the questions selected. Notice that you can not answer more than 15 questions.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Q 1

Solve the following differential equation:

$$\frac{d^2y}{dx^2} - 16y = 0$$

with the initial conditions at x=0:

$$y=1, \quad \frac{dy}{dx} = 12$$

Q 2

Write the following matrix A as the sum of a symmetric matrix R ($R = R^{\mathrm{T}}$) and a skew-symmetric matrix S ($S = -S^{\mathrm{T}}$).
Here, R^{T} and S^{T} mean the transposed matrix of R and S respectively.

$$A = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 0 & -1 \\ -3 & 11 & -4 \end{bmatrix}$$

Q 3

(1) How many positive divisors does '656' have, including 1 and 656 ? Obtain the sum total of the positive divisors of '656'.

(2) Demonstrate that the sum total of positive divisors of $2^{n-1}(2^n - 1)$ is equal to $2^n(2^n - 1)$, if $2^n - 1$ is a positive prime number. The positive divisors include 1 and $2^{n-1}(2^n - 1)$.

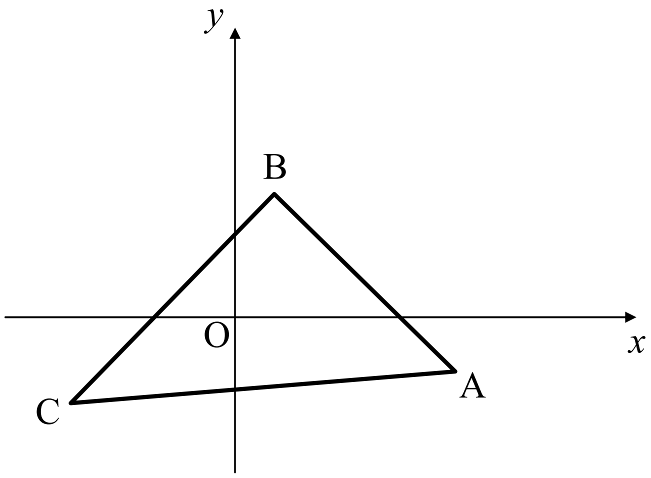
Q 4

When the triangle shown below is transformed by a linear transformation

$$f : \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix},$$

prove that the area of the triangle is invariant under this transformation.

Cautions: *The above-mentioned linear transformation can be interpreted as rotation transformation of the angle θ around the point O. However, the answer "It is clear that area is constant because this transformation is rotation transformation." is not recognized to be a correct answer.*



※ ***Write your answers on next page (answer sheet).***

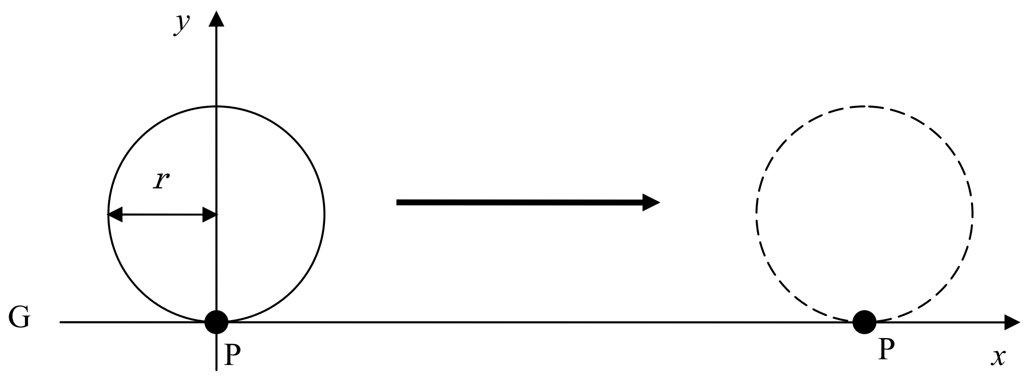
Q 4 (answer sheet)

Q 5

Draw the graph of $f(x) = \left| \dfrac{x^2 - 1}{x^2 + 1} \right|$, and obtain the maximum and minimum values.

Q 6

Consider a circle with a radius of r, which contacts a line G at a point P. When the circle rolls on line G by 360 degrees with no slip, find the area of the region bounded by line G and the locus of P.



Q 7

The sums of three values for all directions are to be identical in this table.
Complete this table.

	14	
		8
	−2	

Q 8

(1) Obtain the volume of a parallel-hexahedron which is made of vectors **a**, **b** and **c** that start at O, with symbols × and ・, which indicate vector and scalar product respectively.

(2) There is a tetrahedron ABCD.
Extend $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ and set points E, F, G, H such that $\overrightarrow{AE} = 2\overrightarrow{AB}$, $\overrightarrow{BF} = 3\overrightarrow{BC}$, $\overrightarrow{CG} = 4\overrightarrow{CD}$, $\overrightarrow{DH} = 5\overrightarrow{DA}$.
Obtain the ratio of the volume of tetrahedron EFGH to that of tetrahedron ABCD.

Q 9

Answer the following questions about a regular polyhedron, which satisfies the relation of $X - Y + Z = 2$ between the number of vertexes (X), sides (Y), and faces (Z).

(1) Show all polyhedrons with your process of reasoning.

(2) Describe the number of vertexes, sides, and faces for each polyhedron.

(3) Determine which of polyhedrons has maximum volume in a circumscribed sphere with a radius of 1. Also describe your process of reasoning.

※ ***Write your answers on next page (answer sheet).***

Q 9 (answer sheet)

Q 10

As shown in figure 1 (a), there is a fixed object L which has a circle of radius 4, inside it. There is a circle M of radius 2 at the center of the circle of radius 4. Five circles (S1 - S5) of radius 1 each are inserted between L and M. If M is rotated, S1 - S5 shall roll, touching both L and M without slip.
If M is rotated counterclockwise as shown in Fig. 1 (b), S1-S5 will rotate clockwise, revolving the surroundings of M counterclockwise. Answer the following questions. Here, five circles (S1-S5) shall roll without touching each other.

(1) How many times does S1 rotate on its axis, during S1 revolves around M ?
(2) How many times does M rotate on its axis, during S1 revolves around M ?

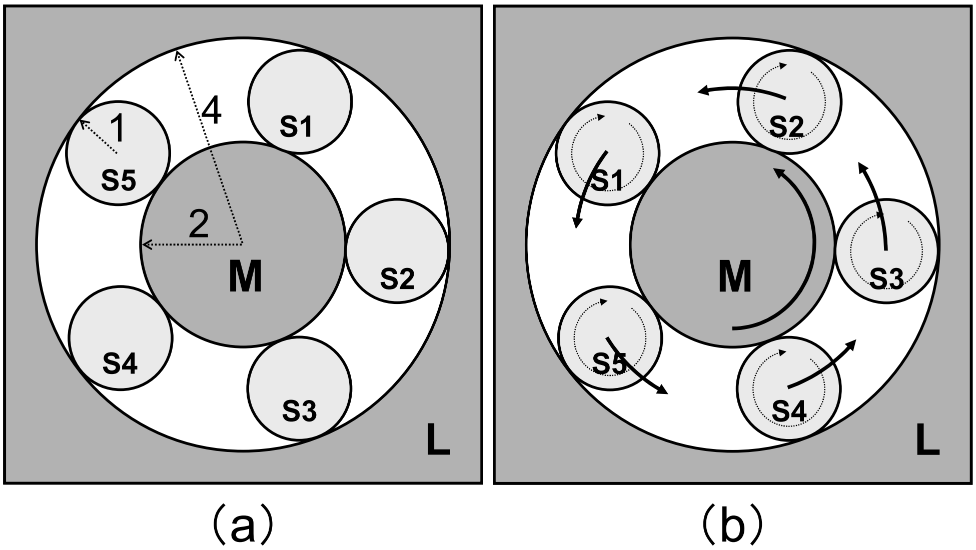


Figure 1

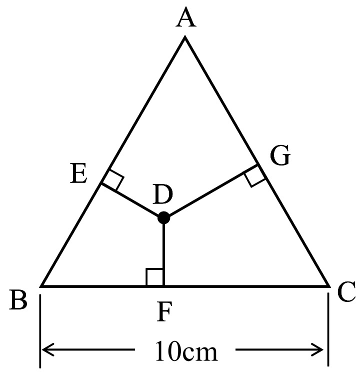
※ *Write your answers on next page (answer sheet).*

Q 10 (answer sheet)

Q 11

The point D can be put on arbitrary position within the equilateral triangle ABC with a side of 10cm. When perpendicular lines are drawn from the point D to the three sides, three cross points of a perpendicular with edges are set to E, F, and G, respectively.

When the point D moves within the equilateral triangle ABC, find the possible range of the sum length of the three perpendicular lines $\overline{DE}+\overline{DF}+\overline{DG}$.



Q 12

Obtain the area of a circle passing through three points (4, -1, 3), (11, 0, 3) and (3, 6, -5).

Q 13

If the angle between the adjacent faces of a regular icosahedron is θ, calculate $cos\theta$.

Q 14

Find out the value of (A) in the following series.

$\pi/4$, $49\pi/36$, $17\pi/36$, $19\pi/12$, $25\pi/36$, (A),

Q 15

The Gregorian calendar is synchronized with the astronomical year by the following rules. Show the orbital period based on these rules (up to fourth decimal place).

- Introduce a leap year in every four years. (February with 29 days)
- Make the last year of every century a common year. (February with 28 days)
- Make the last year of every four centuries a leap year. (February with 29 days)

Q 16

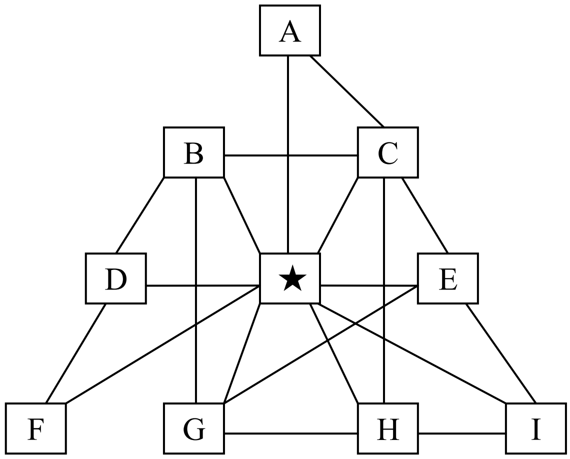
Answer the following questions.

(1) How many “regular octahedron dices”, where each face is marked with one of numbers from 1 to 8 without an overlap, can exist? If one dice becomes the same as another by rotating the dice, they must be regarded as the same ones.

(2) Can a “regular octahedron dice” mentioned in (1), where sum of the numbers of four faces adjacent to any apex is equal to that of another apex, exist? If it can exist, show an example of that “regular octahedron dice”.

Q 17

An airline company has its hub in the city denoted by ★, and lines among 10 cities, ★ and other 9 cities (A, B, C, D, E, F, G, H, I). Show all the routes, which start and end with ★ and visit each city only once. A direction shall have a meaning. For instance, the meaning of A→B is not same with that of B→A.
You do not have to write a graph, but can answer like as follows;
★→A→B→C→D→E→F→G→H→I→★.



Q 18

A company has two products, which requires the combination of inputs as (materials and fuels) shown in the following table.

Products	Inputs: Materials		Inputs: Fuels	Unit Price
	Aluminum	Plastics		
A	1	3	25	¥ 25,000
B	4	4	5	¥ 10,000
Available amounts	100	120	575	

The last row of the table above tells you the available amount of each inputs and the last column shows the unit price. If the company's objective is the maximization of sales, how many products, each product A and B, should the company produce?

Q 19

There are four cards (numbered 1 to 4). Each card has a different picture on its back: diamond, spade, heart, or clover. Three persons (A, B, C) give two testimonies respectively. One of two testimonies is truth, and another is false. Show which picture do these four cards have?

A: Card 2 is not a diamond
Card 1 is not a heart
B: Card 3 is not a spade
Card 1 is a heart
C: Card 1 is a diamond
Card 4 is a clover

Q 20

Every owner of product A and B shall buy and replace a new product as follows. The owner of the product A chooses A with probability α $(0 < \alpha < 1)$, and B with probability $1-\alpha$. The owner of the product B chooses B with probability β $(0 < \beta < 1)$, and A with probability $1-\beta$. Answer the following questions.

(1) What is the probability that owner who possesses the product A at first has the product B after 2nd change?

(2) What is the probability that owner who possesses the product A at first has the product B after n-th change?