Examinee's No. ____________________

Department of Systems Innovation,
School of Engineering, The University of Tokyo
2011 Entrance Examination and Answer Sheets

Mathematics

Monday, August 30, 2010
13:00－15:30

Notice

1. Do not open this booklet until the start of the examination.

2. If you find missing or badly printed pages, please raise your hand to let the supervisor know.

3. Write your examinee's number on top of this sheet. Do not write your examinee's number or your name anywhere else in this booklet.

4. Write the answer including the outline of your solution below the question.

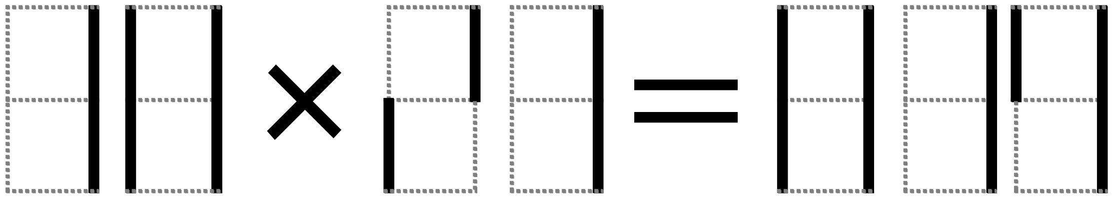
5. Sheets for calculation are distributed separately.

6. There are 20 questions in total. Choose and answer 15 questions out of 20.
Check the question numbers of the questions you have selected in the bottom row. Notice that you cannot answer more than 15 questions.

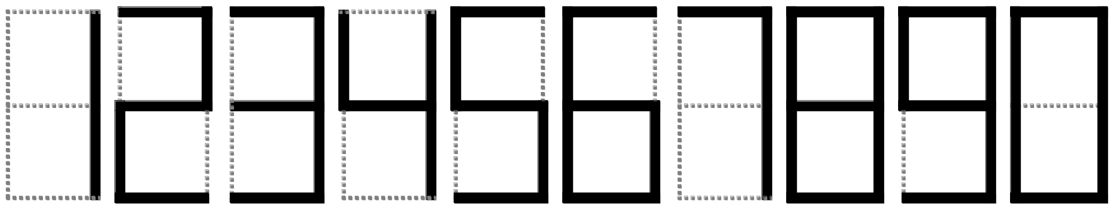
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Q.1

The following figure shows a calculating equation on a calculator. The display of this calculator is partly out of order as the horizontal lines are not displayed at all, while all vertical lines of figures are displayed. Find the numbers of the calculating equation.



The figures in their normal condition are displayed as follows.



Q.2

There are 100 students in the master course of a department. There are 70 Japanese students and 30 international students. Out of 100, 60 are male students. The number of female international students is larger than that of male international students.

(1) Obtain the possible range of the number of female Japanese students.

(2) 60 out of 100 students take course A and at least half of them are female students. Obtain the possible range of the number of Japanese students taking this course.

Q.3

What is the n-th power of the following matrix A? Here, n is a positive integer.

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

Q.4

When x is divided by 2, the remainder is 1.

When x is divided by 3, the remainder is 2.

When x is divided by 4, the remainder is 3.

When x is divided by 5, the remainder is 4.

Find the minimum natural number x.

Q.5

Using mathematical induction, prove that the following relation holds for any positive integer n.

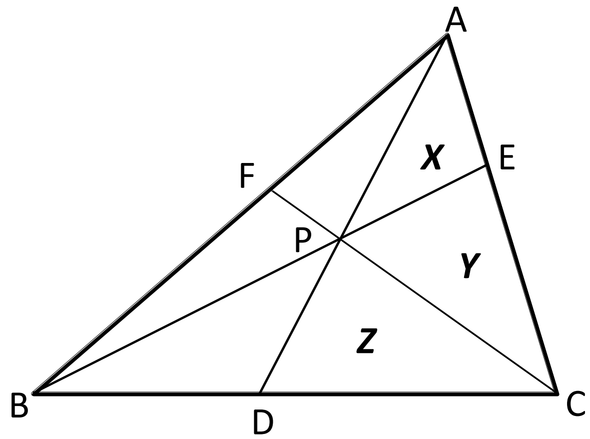
$$2^n < \frac{3^n}{n}$$

Q.6

There is a circular cone in which the height is 3 and the radius of the base circle is $\sqrt{3}$. A sphere is inscribed in this cone. Obtain the ratio of the volume of the cone to the volume of the sphere.

Q.7

As shown in the figure, point P is in a triangle ABC. D is an intersection of the extended AP and BC. E is an intersection of the extended BP and CA. F is an intersection of the extended CP and AB. When the areas of triangles APE, EPC and CPD are X, Y and Z respectively, express the area of the triangle ABC with X, Y, and Z.



Q.8

Differentiate $f(x)$ with respect to x, where x is a positive real number.

$$f(x) = x^x$$

Q.9

Find the vector $\mathbf{c}$ which is perpendicular to both $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, and whose magnitude is 3. Here $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ denote unit vectors along the x, y, and z axes.

Q.10

Find the 5-digit integer number ABCDE. ABCDE is calculated by the following addition of two 4-digit integer numbers, ADCF + GBHD. Each alphabet corresponds to a different integer number from 0 to 9.

	A	D	C	F
+	G	B	H	D
A	B	C	D	E

Q.11

An examination for a certain disease V results either in positive or negative. In case that the result is positive, the patient is strongly suspected to be affected by the disease V. But on rare occasions the result turns out to be positive even if the patient is not affected by V. On the other hand, there are also some rare occasions in which the result turns out to be negative even if the patient is affected by V.

Calculate the probability that one is affected by V when the examination result is positive. Assume the probability that the result becomes positive when the patient is affected by V is 90%, and the probability that the result becomes negative when the patient is not affected by V is 80%. The fraction of people affected by V is 1 out of 5.

Q.12

n is a positive integer. List all the possible figures that can appear in the last digit of n^{100} when written in decimal notation.

Q.13

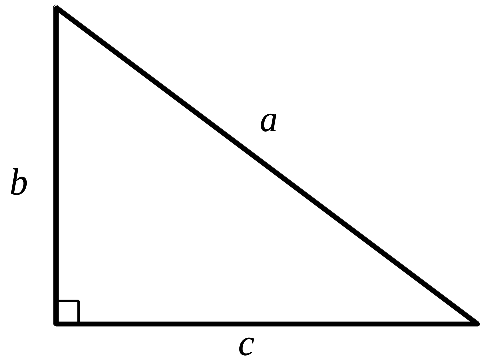
Oxygen atoms have three isotopes, ^{16}O, ^{17}O, and ^{18}O, whose natural abundances (number ratios) are 99.76%, 0.04% and 0.20%, respectively. When oxygen atoms are combined randomly, calculate the percentage of oxygen molecule O_2 having the mass of 34.

Q.14

Obtain the length of the curve in the xy – plane, $y = \log_e(1 - x^2)$, between x is -0.5 and 0.5.

Q.15

Consider a right triangle with legs b and c, and hypotenuse a, where a, b, and c are natural numbers. Prove that either b or c must be an even-number.

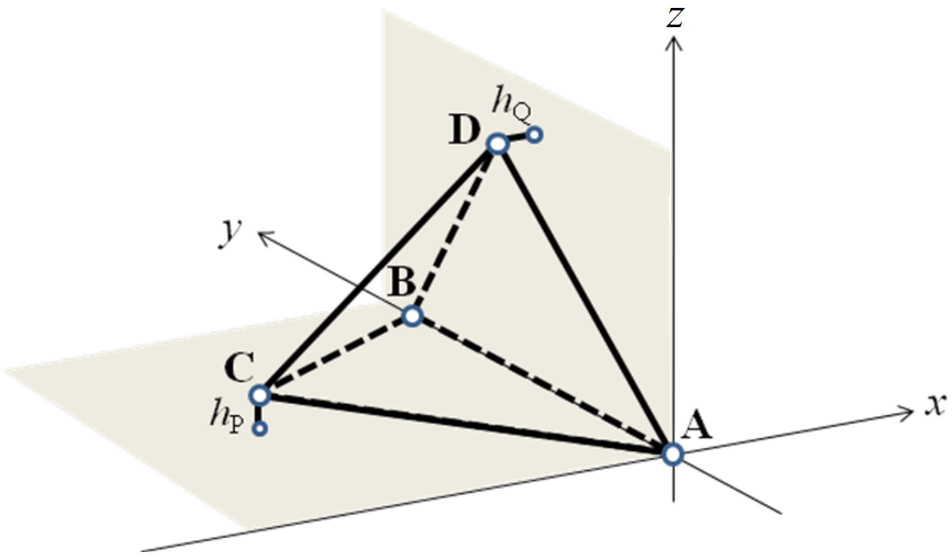


Q.16

A lottery ticket costs 300 yen. One of the numbers from 1 to 4 is written on the ticket. You can get goods depending on the number. There are four kinds of goods. Find the expected cost to obtain all the four kinds of goods. A lottery is equally probable in terms of the number on it, which does not change if drawing is repeated.

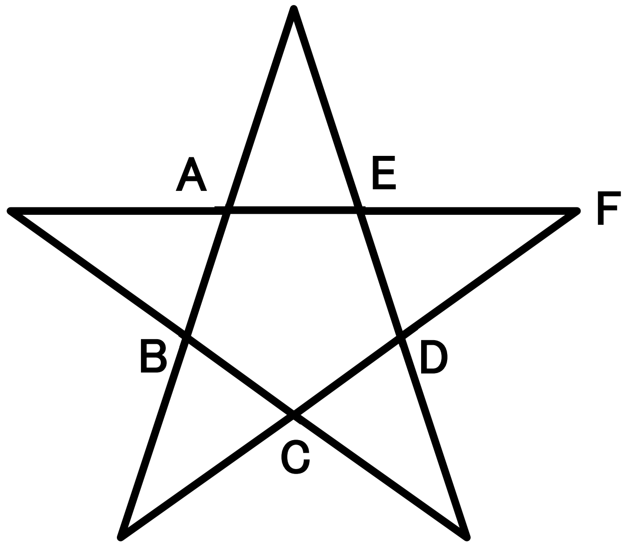
Q.17

Suppose that the edge AB of a regular tetrahedron ABCD of the edge-length 1 lies on the $y-$axis as shown in the figure. Let h_P be the length between the vertex C and the projection of C on the $xy-$plane, and h_Q be the length between the vertex D and the projection of D on the $yz-$plane. Obtain the length of h_P when $h_P = h_Q$.



Q.18

Consider the star shape shown in the figure, which is formed by extending each side of a regular pentagon ABCDE. The side length of the pentagon is 1. Lines AE and CD cross at F.　Find the length of segment EF.



Q.19

Before crossing a street, pedestrians have to wait from 1 to 5 minutes.

The following equation gives the probability density function of the waiting time x minutes on the interval [1, 5].

$$f(x) = \frac{5}{4x^2}.$$

What is the probability that a pedestrian has to wait more than 2 minutes before crossing the street?

Q.20

Two variables x and y are linked by the following equation.

$$x^2 - 2xy + y^3 = 1$$

Obtain the derivative dy/dx at $x = 2$ and $y = 1$.